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ECOSYSTEMES CONTINENTAUX  
ECCOREV  
ET RISQUES ENVIRONNEMENTAUX



## Bayesian identification of parameters for modelling chloride ingress into reinforced concrete

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# 1. Introduction

Corrosion in reinforced concrete(RC) structures :

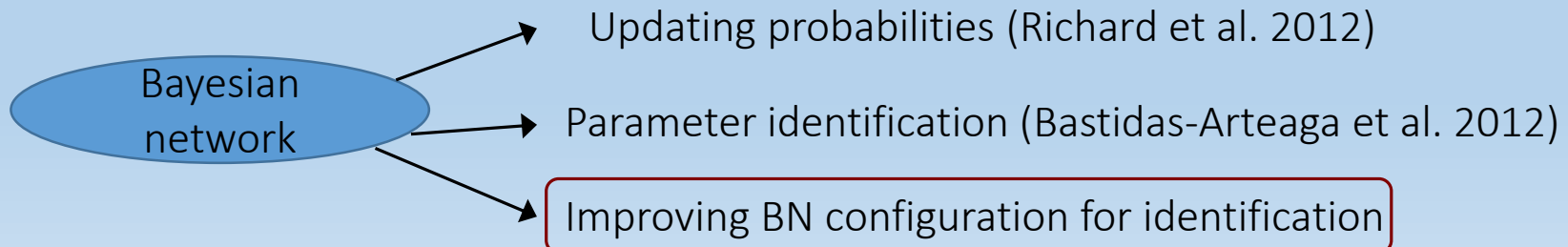
- Shorten the lifetime of RC structures
- Generate important damages after 10 or 20 years (Rosquoët et al. 2006; Bonnet et al. 2009)

→ Inspection to determine concentration level of chloride is important to:

- Minimizing the risks and consequences of corrosion initiation
- Ensuring optimal levels of serviceability and safety



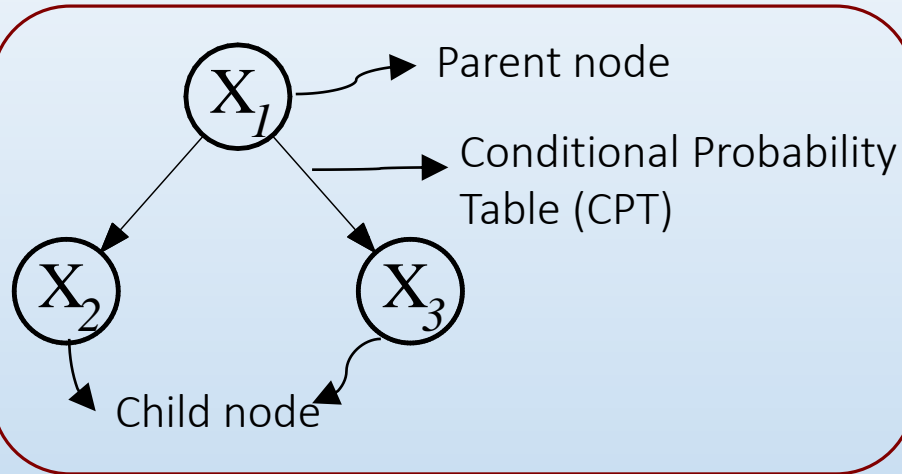
Comprehensive parameter identification from limited inspection data → important for optimizing maintenance costs and improving lifetime assessment



1. Introduction
2. Bayesian identification and its application to chloride ingress
3. Influence of BN configuration on identification
  - 3.1 General aspects for comparison of BN configurations
  - 3.2 Identification using one point in depth of inspection
  - 3.3 Identification using full depth of inspection
4. Assessment of the probability of corrosion initiation
5. Conclusions and perspectives

## 2. Bayesian identification and its application to chloride ingress

### Bayesian network (BN)



### Chloride ingress model

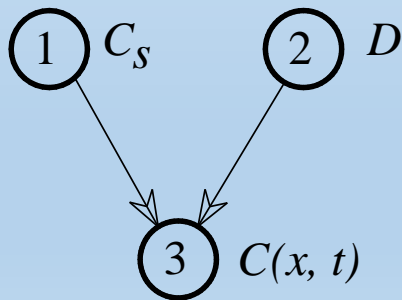
[Tuutti 1982]

$$C(x, t) = C_s \left[ 1 - \operatorname{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \right]$$

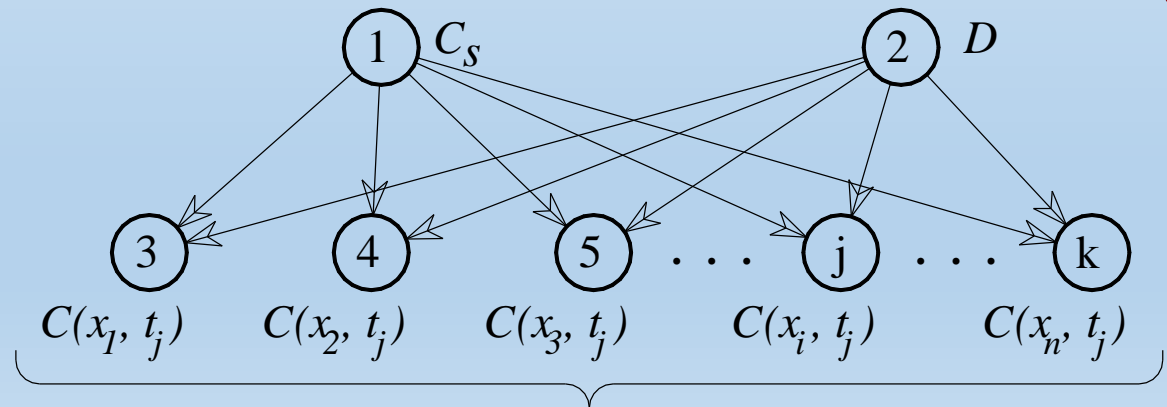
With:

- $C_s$ : chloride surface concentration ( $\text{kg}/\text{m}^3$ )
- $D$ : effective chloride diffusion coefficient ( $\text{m}^2/\text{s}$ )

### The BN apply to chloride ingress



From one point in depth



From  $n$  point in depth

## 2. Bayesian identification and its application to chloride ingress



Assuming that  $C_s$  and  $D$  are independent:  $p(D, C_s) = p(D) \cdot p(C_s)$ , the probability  $p(C(x, t))$  can be calculated as follow (Nguyen. 2007):

$$p(C(x, t)) = \sum_{D, C_s} p(C(x, t) | D, C_s) p(D, C_s)$$

Introducing some evidences from chloride profiles collected,  $p(C(x, t) | o)$ , the posterior distribution can be computed by applying Bayes' theorem:

$$p(D | o) = p(D | C(x, t)) p(C(x, t) | o) \quad \text{with} \quad p(D | C(x, t)) = \frac{p(C(x, t) | D) p(D)}{p(C(x, t))}$$

$$p(C_s | o) = p(C_s | C(x, t)) p(C(x, t) | o) \quad \text{with} \quad p(C_s | C(x, t)) = \frac{p(C(x, t) | C_s) p(C_s)}{p(C(x, t))}$$

All the determination of these conditional probabilities are carried out by the Bayesian Network Toolbox (BNT) (K.P. Murphy) on the Matlab® Software

## 3.1 General aspects for comparison of BN configurations

### Discretization of parameters

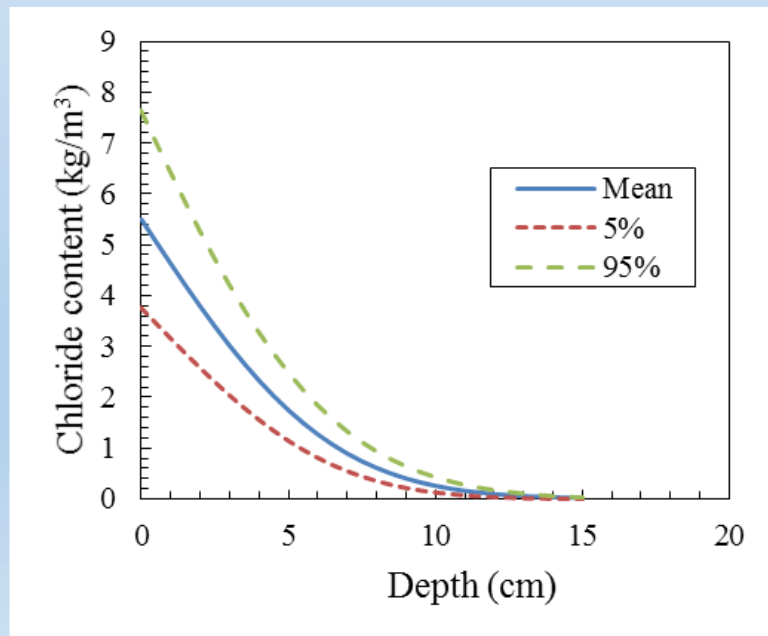
Parameters	Number of intervals	Priori distribution	Range
$C_s$ (kg/m <sup>3</sup> )	16	Uniform	( 1; 17)
$D$ (m <sup>2</sup> /s)	20	Uniform	(2e-12 ; 8e-12)
$C(x,t)$ (kg/m <sup>3</sup> )	-	-	(0 ; 17)

### Theoretical values for generating numerical evidences (Vu & Stewart 2000)

Parameters	Distribution	Mean	Standard deviation
$C_s$	Lognormal	5.5 (kg/m <sup>2</sup> )	1.2(kg/m <sup>2</sup> )
$D$	Lognormal	4e-12 (m <sup>2</sup> /s)	5E-13(m <sup>2</sup> /s)

MCS

10000 profiles  
at  $t_{ins} = 10$  years



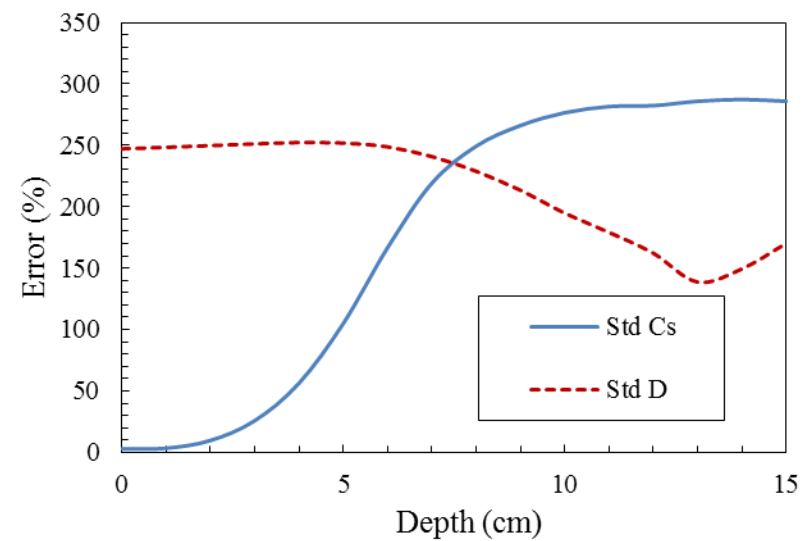
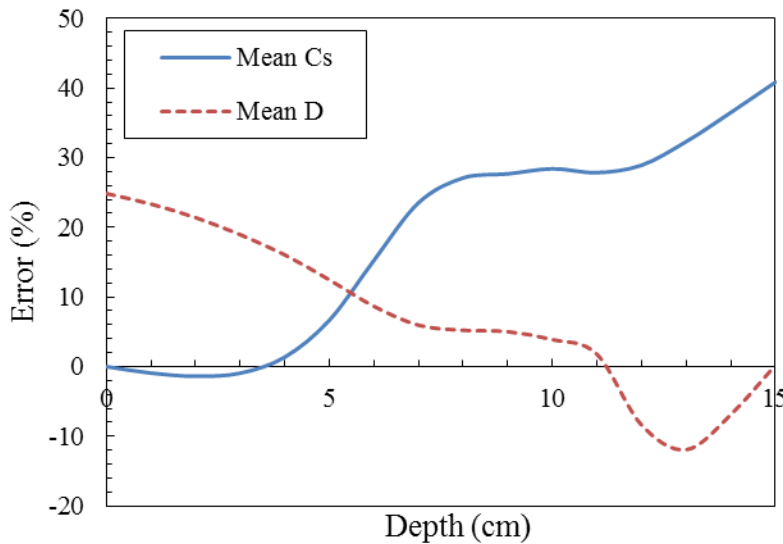
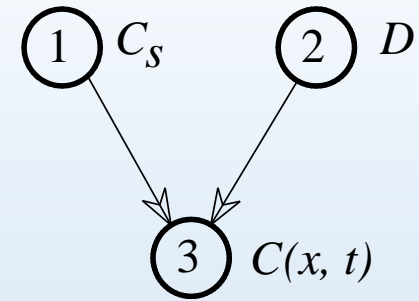
Evaluating configurations

$$Error(y) = \frac{y_{identified} - y_{theory}}{y_{theory}} . 100\%$$

$y$  : represents for the mean or the standard deviation of parameters

## 3.2 Identification using one point in depth of inspection

- Inspection time  $t_{\text{ins}} = 10$  years
- The BN consists of three nodes:  $C_s$ ,  $D$  and  $C(x,t)$



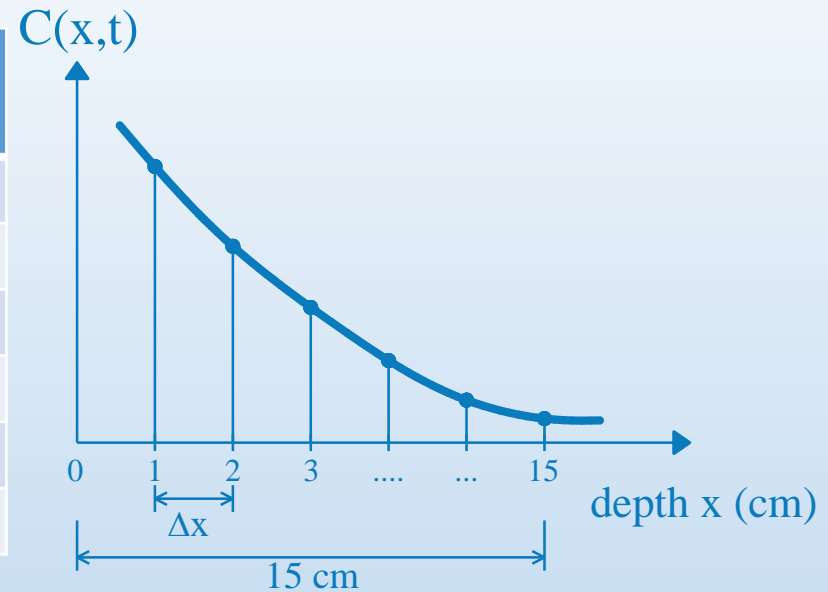
### Results:

- The chloride concentration at the surface is most valuable in the identification of  $C_s$ .
- The chloride content at deeper parts reduces the error on the identification of  $D$ .
- However, using evidence from one point in depth could not give a good estimation for  $D$ .

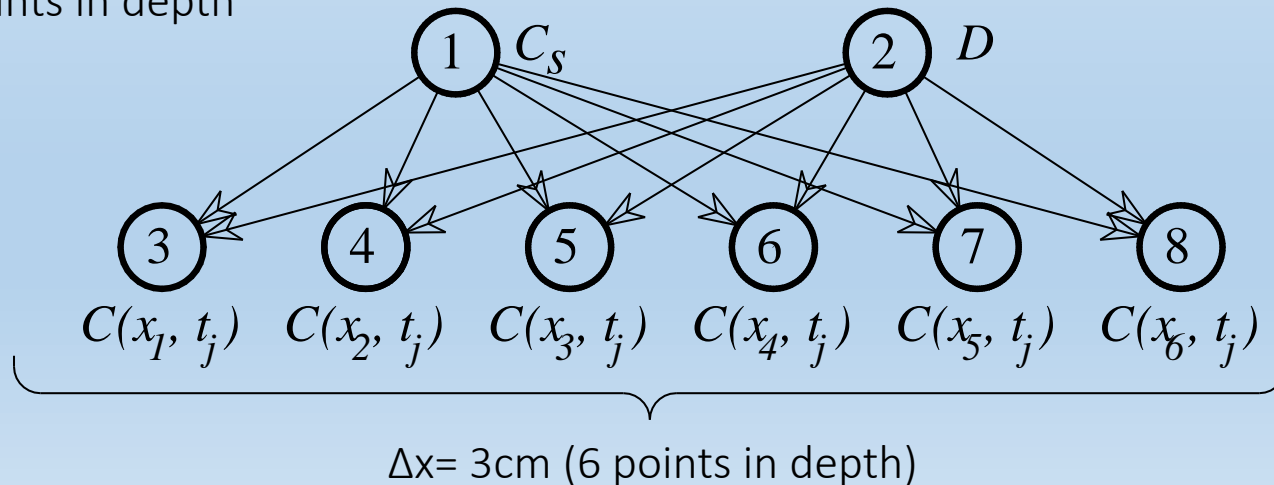
### 3.3 Identification using full depth of inspection

#### Discretization the inspection depth

Case	$\Delta x$ (cm)	Discretization	Number of points in depth
1	3	0:3:15	6
2	2	0:2:15	8
3	1	0:1:15	16
4	0.5	0:0.5:15	31
5	0.3	0:0.3:15	51
6	0.2	0:0.2:15	76



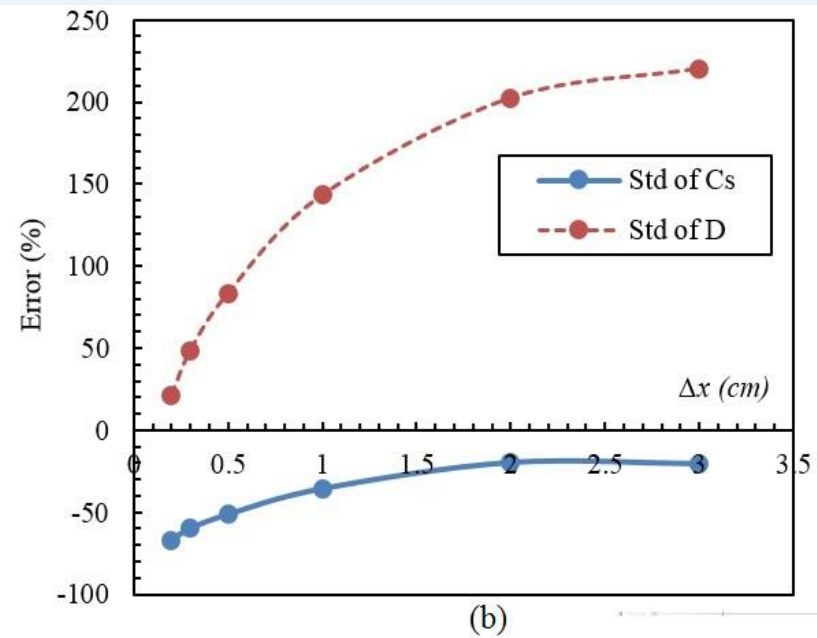
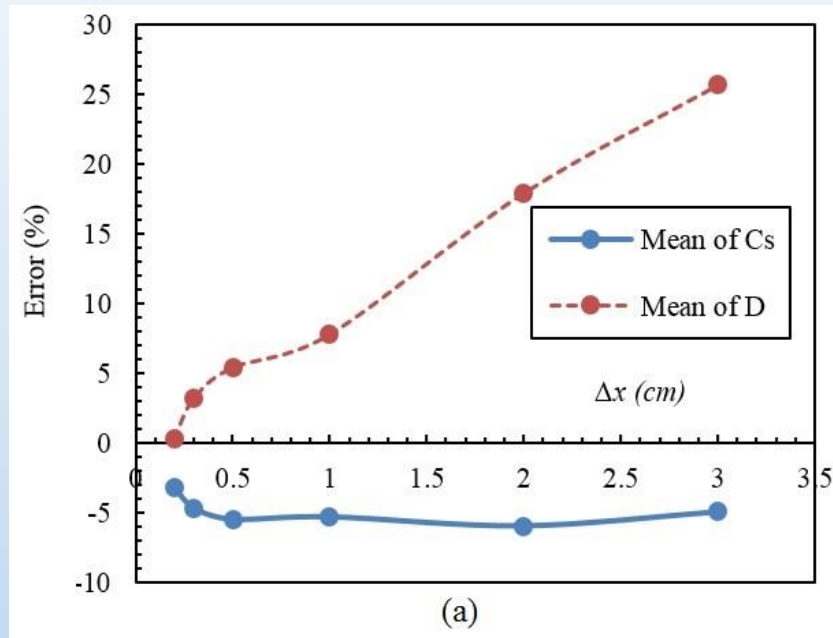
- Inspection time  $t_{\text{ins}} = 10$  years
- The BN consists of :  $C_s$ ,  $D$  and  $n$  child nodes  $C(x,t)$  corresponding to number of points in depth





### 3.3 Identification using full depth of inspection

#### Errors in identification using full depth of inspection



#### Results:

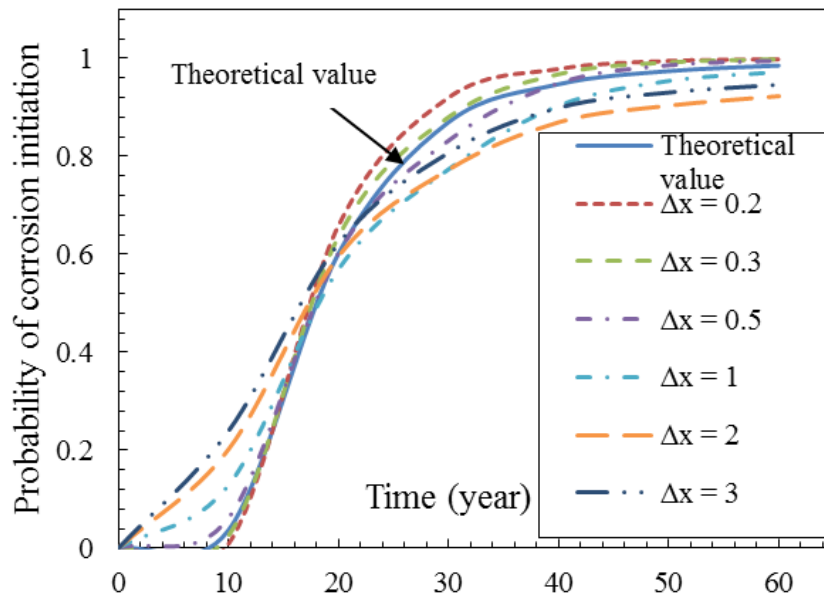
- Using several measurements could not improve the identification of Cs.
- Discretizing the inspection depth into small intervals could give a better estimation for D.

## 5. Assessment the probability of corrosion initiation

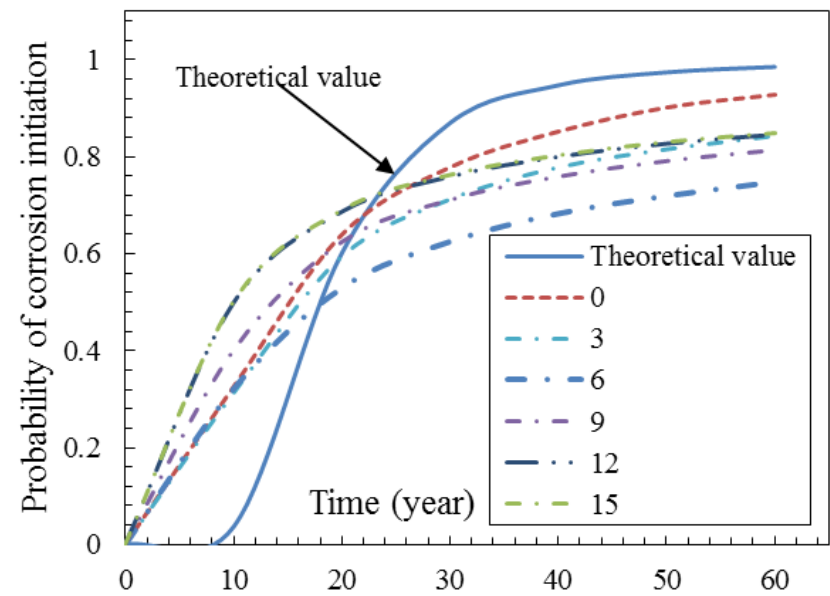
Probability of corrosion initiation with sufficient data:

a. From full inspection depth

b. From single point inspection depth



(a)



(b)

### Results:

- $\Delta x$  is small  $\rightarrow$  the prediction results are more close to the target.
- Using data from one depth point  $\rightarrow$  the predictions are unsatisfied

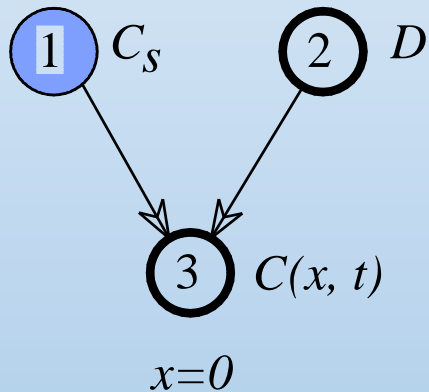
## 5. Assessment the probability of corrosion initiation

Improving the prediction of probability of corrosion initiation with limited data:

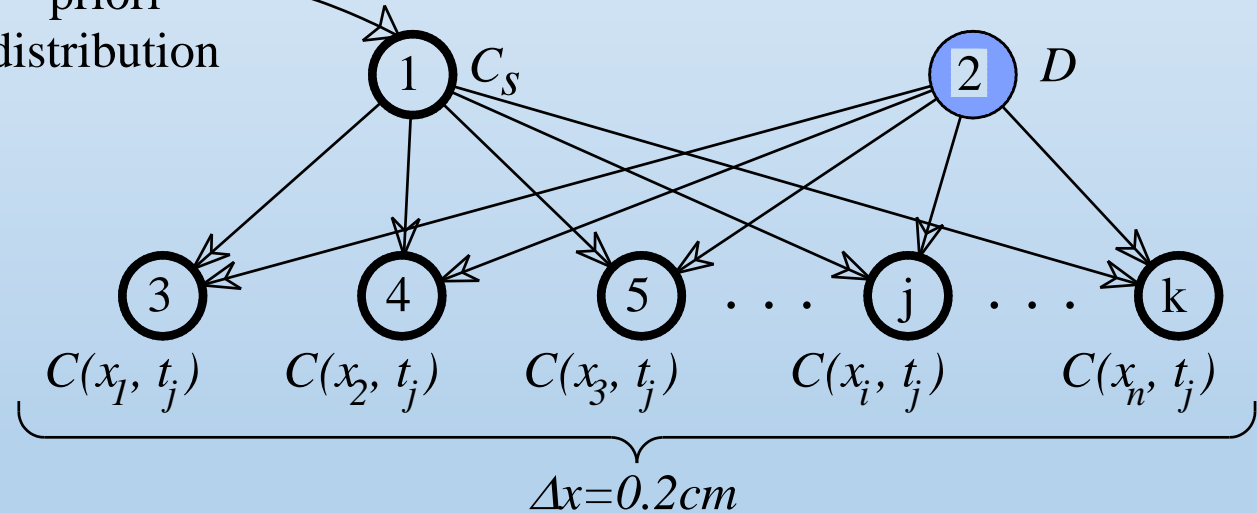
- 15 chloride profiles
- Improving:

Step 1

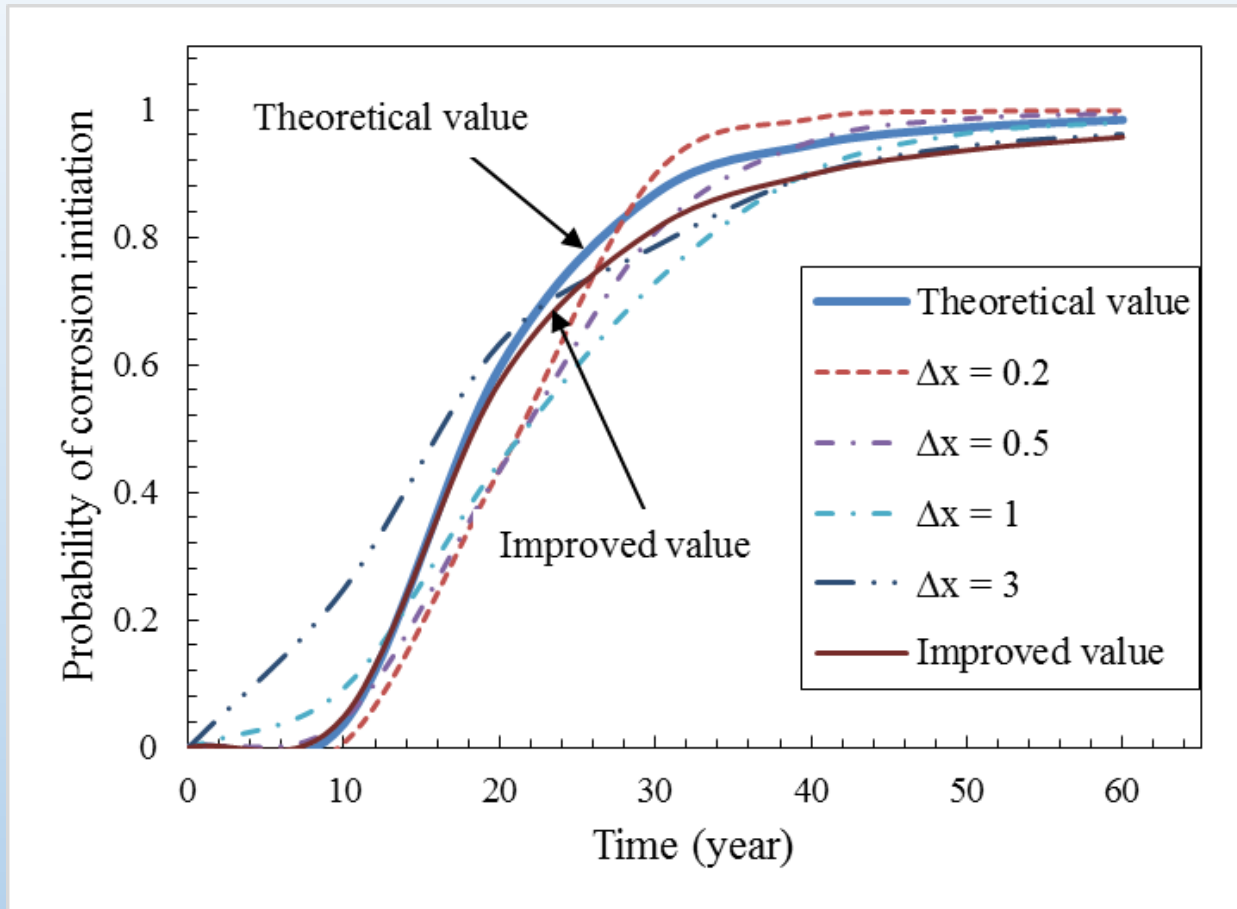
Step 2



priori distribution



## 5. Assessment the probability of corrosion initiation



### Results:

- With limited data, improved value is more close to the target

### Conclusions:

- Bayesian networks could provide a probabilistic model for identification random parameters from inspection data
- Each parameter corresponding to a configuration providing the best estimation:
  - + For Cs: using evidences from  $x \approx 0$
  - + For D: using evidences from small  $\Delta x$
- With limited data, an improved procedure could give a prediction closing to the target.

### Perspectives:

- Dealing with different inspection times
- Application to real data
- Extension to other chloride ingress models
- Consideration of measurement errors.

END

THANK YOU FOR YOUR ATTENTION