A Bayesian Network framework for probabilistic identification of model parameters from normal and accelerated tests: application to chloride ingress into concrete

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Seminar GeM
15 Sept 2016

❑ Challenges in maintenance strategies

Corrosion

- Shorten the lifetime of reinforced concrete (RC) structures
- Important damages after **10-20 years**

(William (2014))

Maintenance: Periodical inspection every Δt year

Model parameters (C_s, D, n_D, etc)

← Inspection data

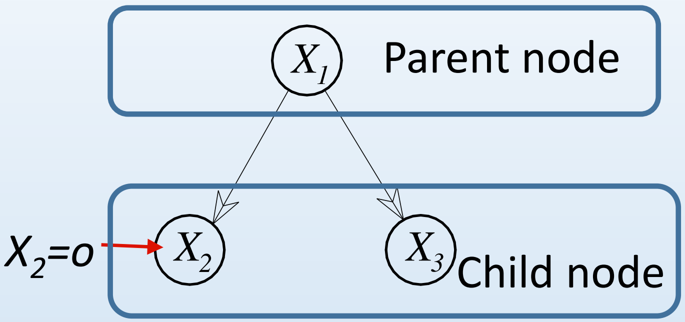
← Chloride ingress models

- **Limited:**
  - **inspection techniques,**
  - **time-consuming**
- **Uncertainty:**
  - **model parameters,**
  - **measurements**

Parameter identification

- Integrate **all uncertainties** for parameter identification ?
- **Improve/optimise** the identification with **limited data** ?
- Characterise the **mid- and long-term** behaviour of material ?

Theory of BN



The joint Probability Mass Function

$$P(X_1,X_2,X_3)=P(X_1)P(X_2|X_1)P(X_3|X_1)$$

Update with evidence $X_2 = o$:

$$P(X_1,X_3|o)=\frac{P(X_1,o,X_3)}{P(o)}=\frac{P(X_1)P(o|X_1)P(X_3|X_1)}{\sum_{X_1}P(X_1)P(o|X_1)}$$

Chloride ingress modelling

Time-independent

$$C(x,t)=C_s\left[1-erf\left(\frac{x}{2\sqrt{D.t}}\right)\right]$$

Collepardi et al. (1972)

Time-dependent

$$C(x,t)=C_s\left[1-erf\left(\frac{x}{2\sqrt{\frac{D_0}{1-n_D}\left[\left(1+\frac{t'_{ex}}{t}\right)^{1-n_D}-\left(\frac{t'_{ex}}{t}\right)^{1-n_D}\right]\left(\frac{t'_0}{t}\right)^{n_D}t}}\right)\right]$$

Nilsson and Carcasses (2004)

BN application to chloride ingress

Chloride content: $C(x_i,t_j)=f(x,t,C_s,D,n_D,..)$

Child nodes $n=n_x n_t$

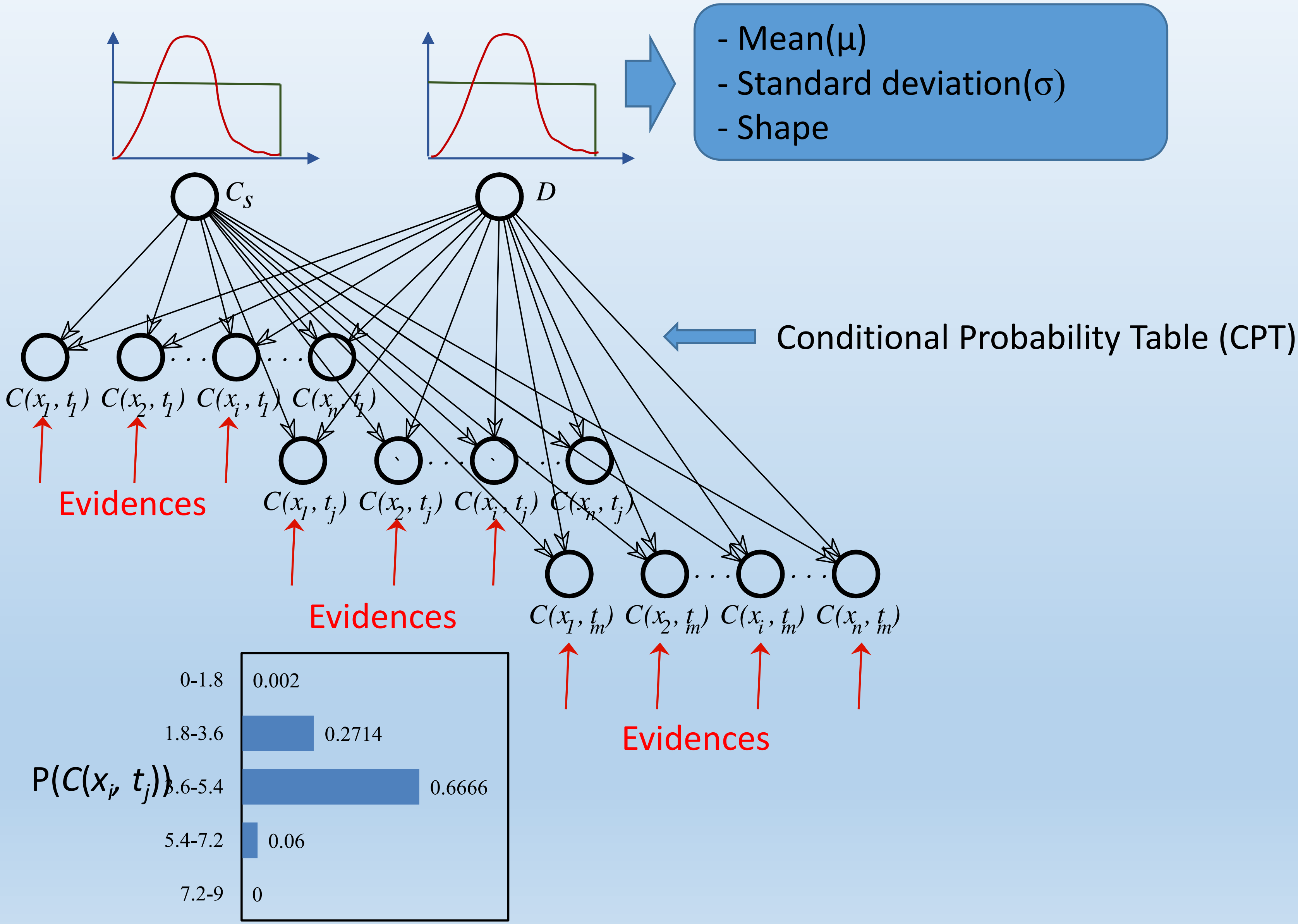
Parent nodes → Depend on selected model

Number of **points in depth**

Number of **inspection times**

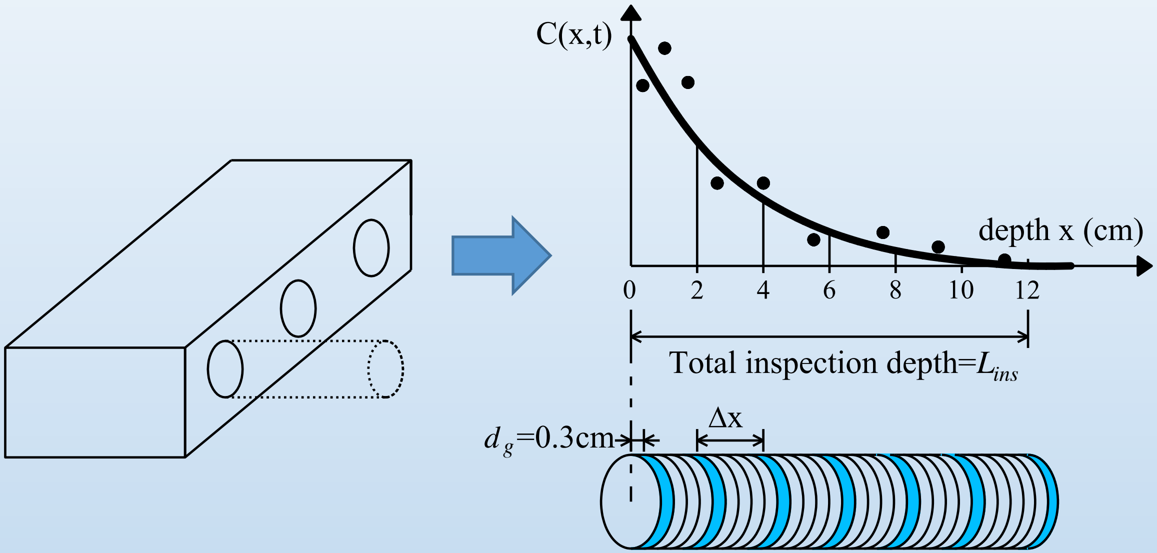
$$C(x,t)=C_s\left[1-erf\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

BN application to chloride ingress



❑ Evidences

Real chloride profiles

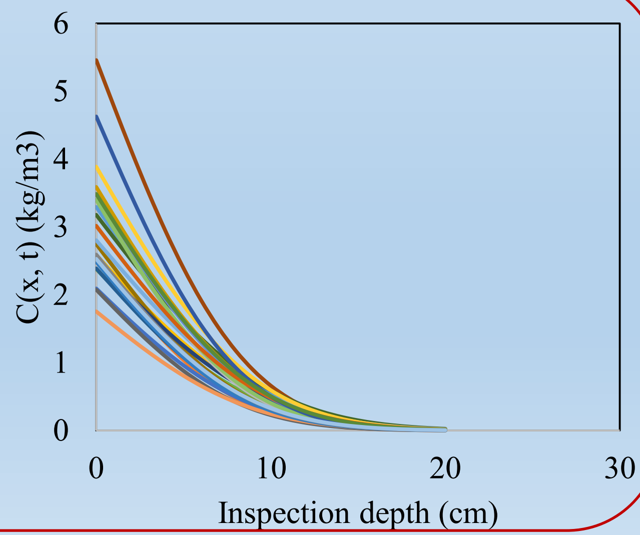


Simulated chloride profiles

Given model parameters:

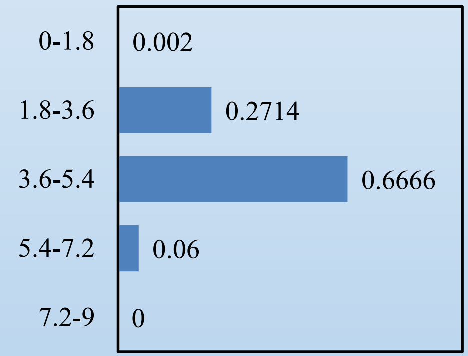
$C_s \sim$ LN (2.95; 0.59)

$D \times 10^{-12} \sim$ LN (7.05; 1.05)

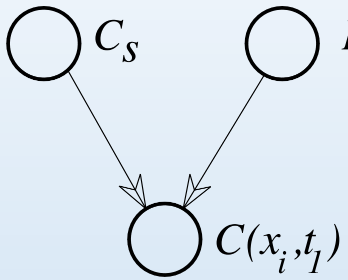


Evidences in BN

$P(C(x_i, t_j))$

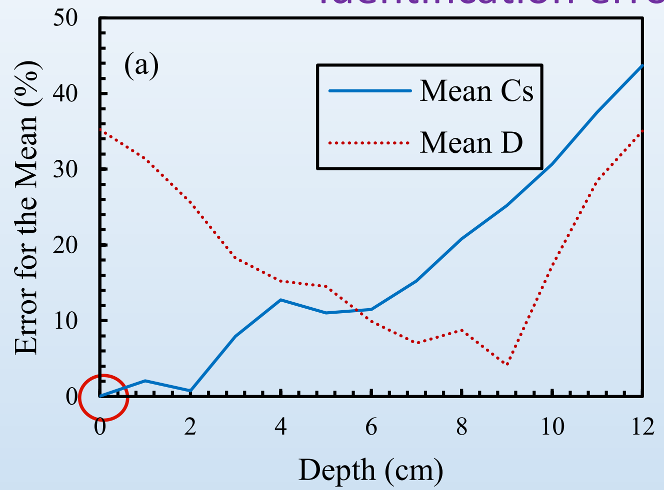


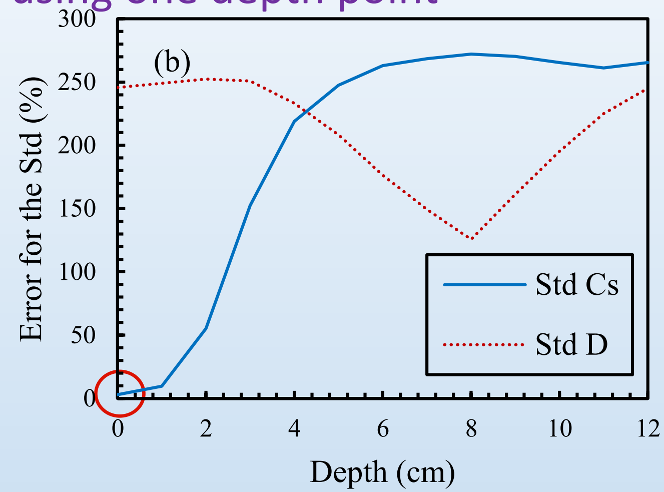
❑ Identification using one inspection point



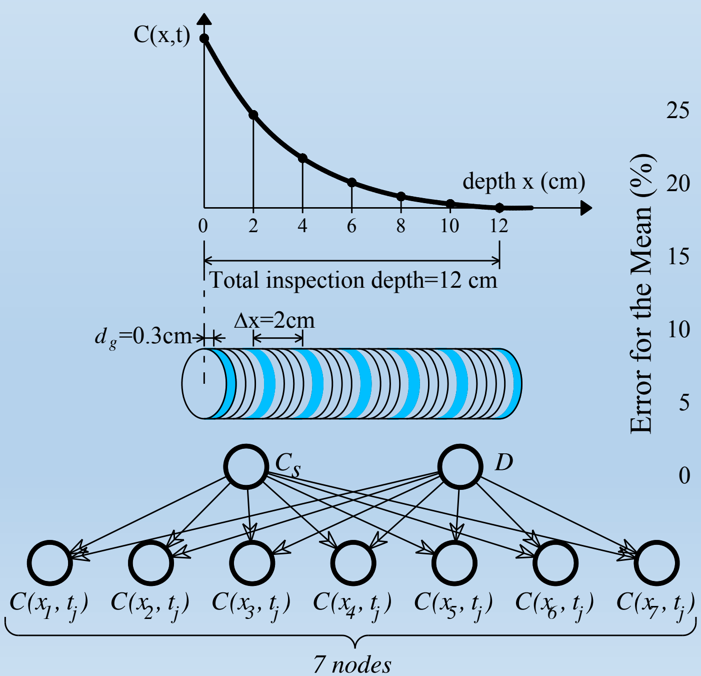
Identify Cs with C(x=0;t)

Identification error using one depth point

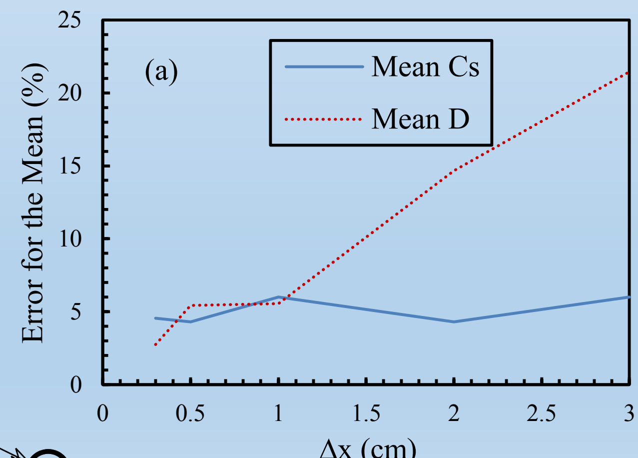


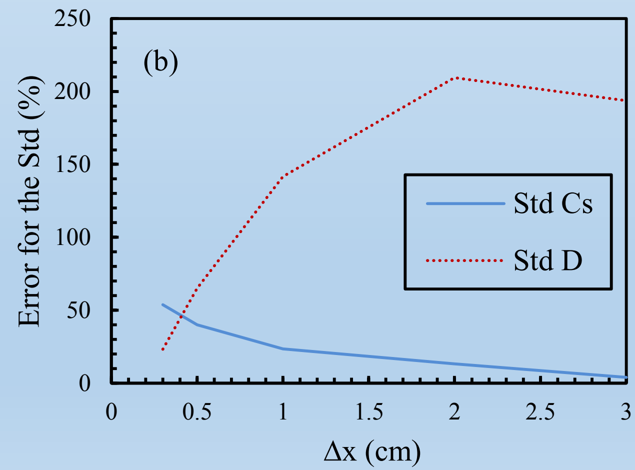


❑ Identification using several inspection points



Identification error using several depth points





Identify D → using small Δx

❑ Probability of corrosion initiation

- The limit state function:

$$g(\mathbf{X},t) = C_{th}(\mathbf{X}) - C_{tc}(\mathbf{X},t)$$

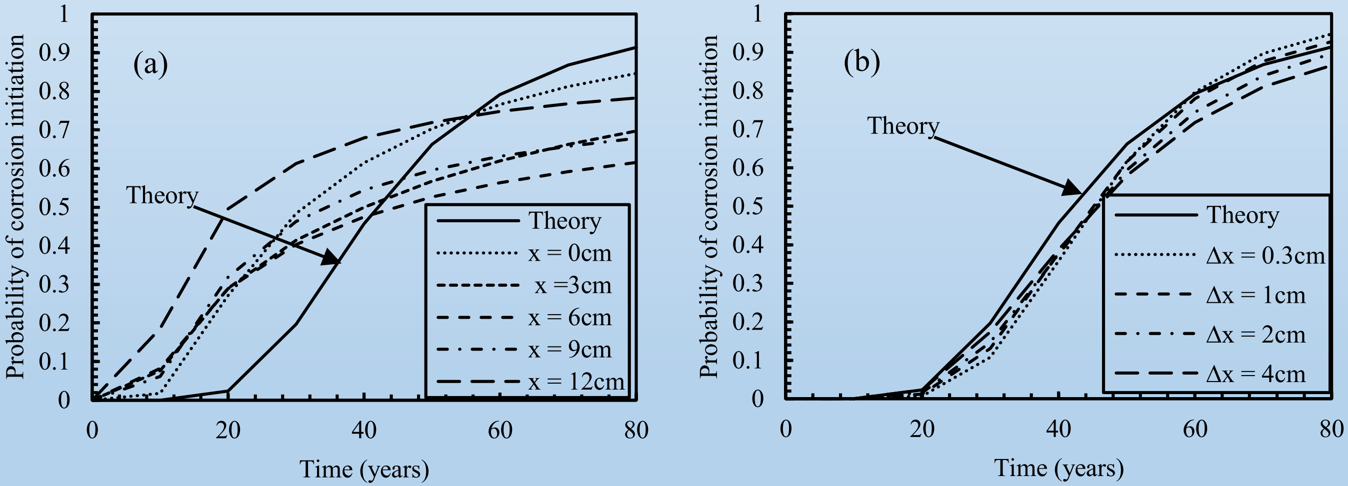
Threshold value (C_{th})

Chloride concentration at cover depth (C_{tc})

- The probability of corrosion initiation:

$$p_{ini}(t) = P\left(g(\mathbf{X},t) \le 0\right) = \int_{g(\mathbf{X},t)\le 0} f_{\mathbf{X}}(x)\,dx_1 \ldots dx_n$$

➢ Assessment of Pini with larger data



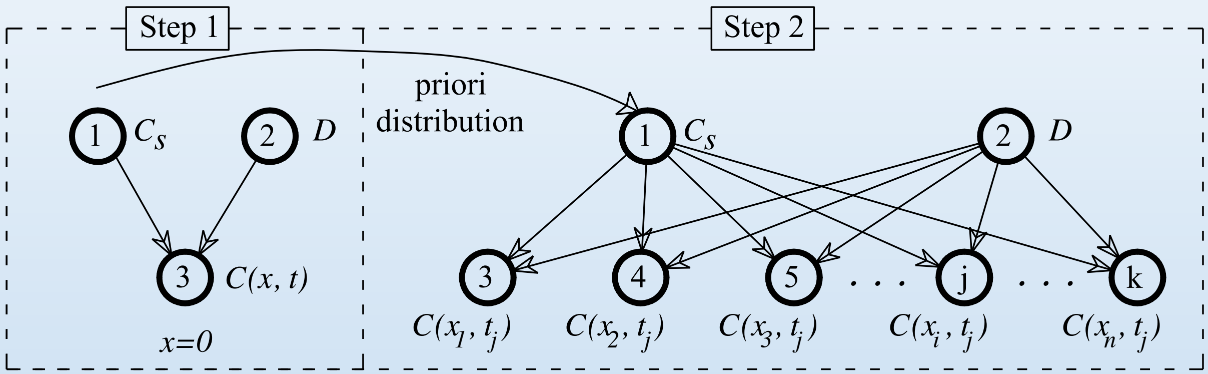
Limited data ?

-One inspection point → unsatisfied predictions

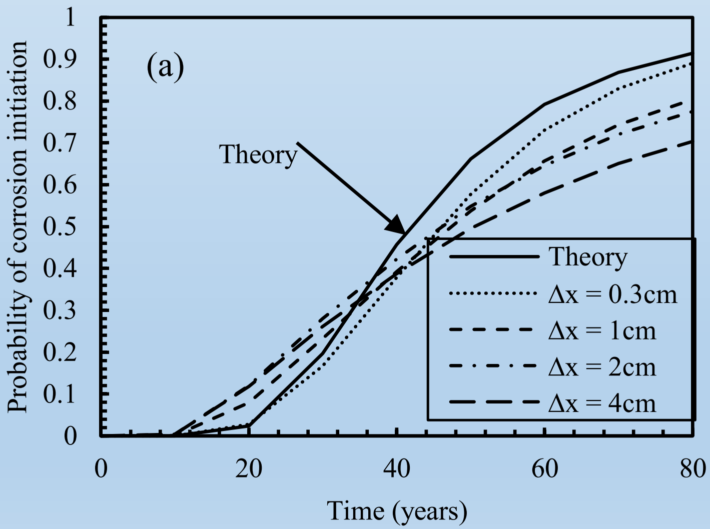
-Δx is small → close to theory

❑ Asessment of P_{ini} from limited data and improvement approach

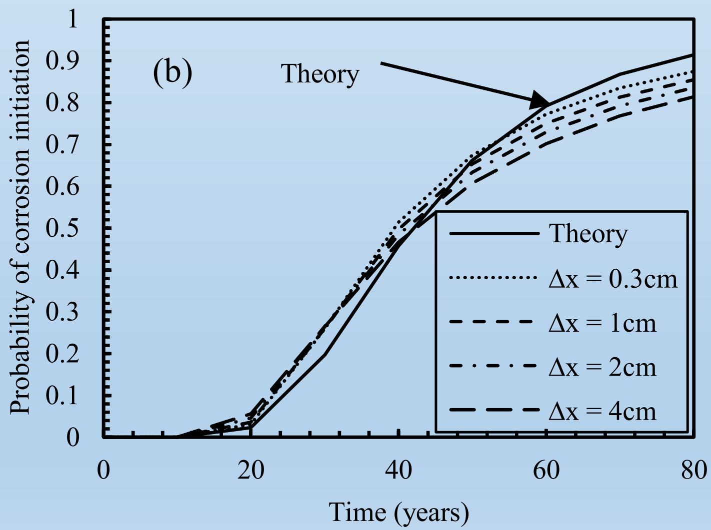
Improvement procedure



15 chloride profiles



15 chloride profiles - Improvement



Better assessment of P_{ini}

❑ Experimental setup - description

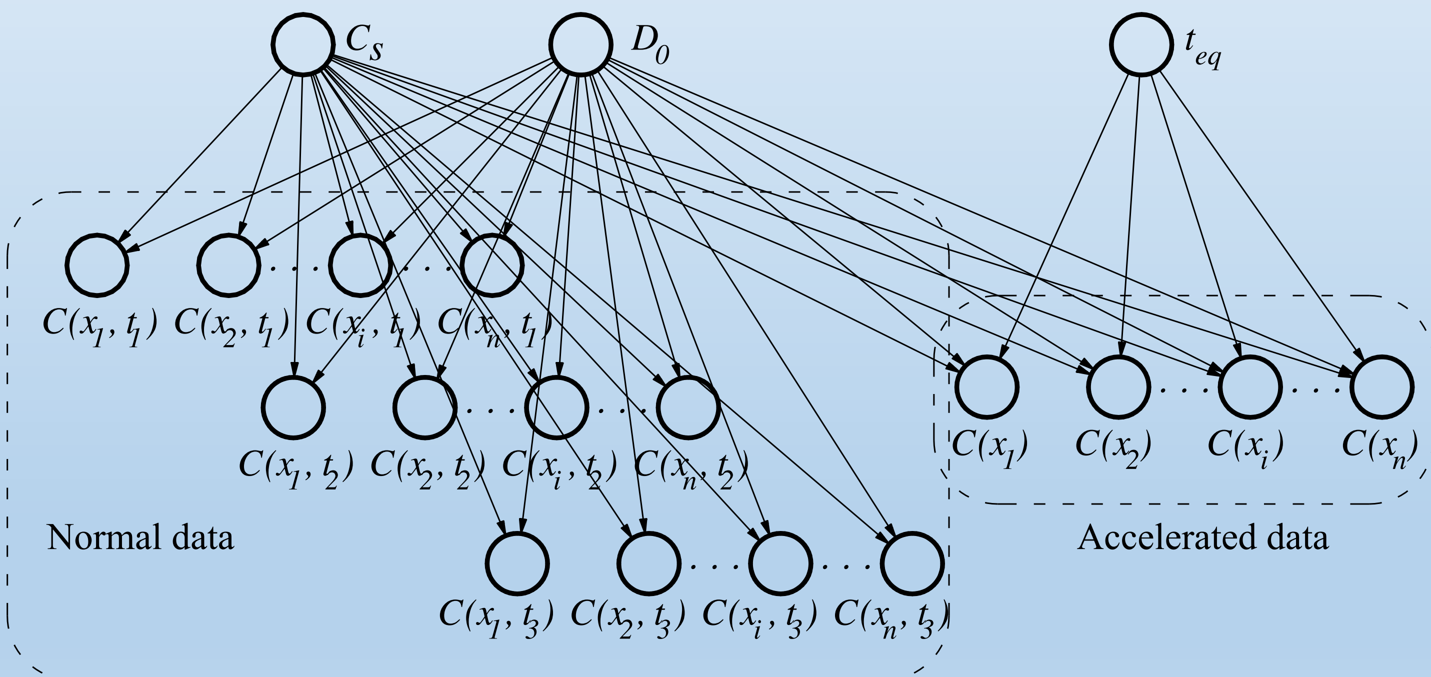
Normal test	Accelerated test
- Characterise the time-dependency of chloride ingress mechanisms	- Characterise mid- and long-term chloride ingress mechanisms
- Slow process → require significant time	- Faster but **equivalent exposure time is unknown (*t_{eq}*)**

Determine equivalent time to use information of accelerated tests for identification purposes?

❑ BN modelling: using time-dependent chloride ingress model

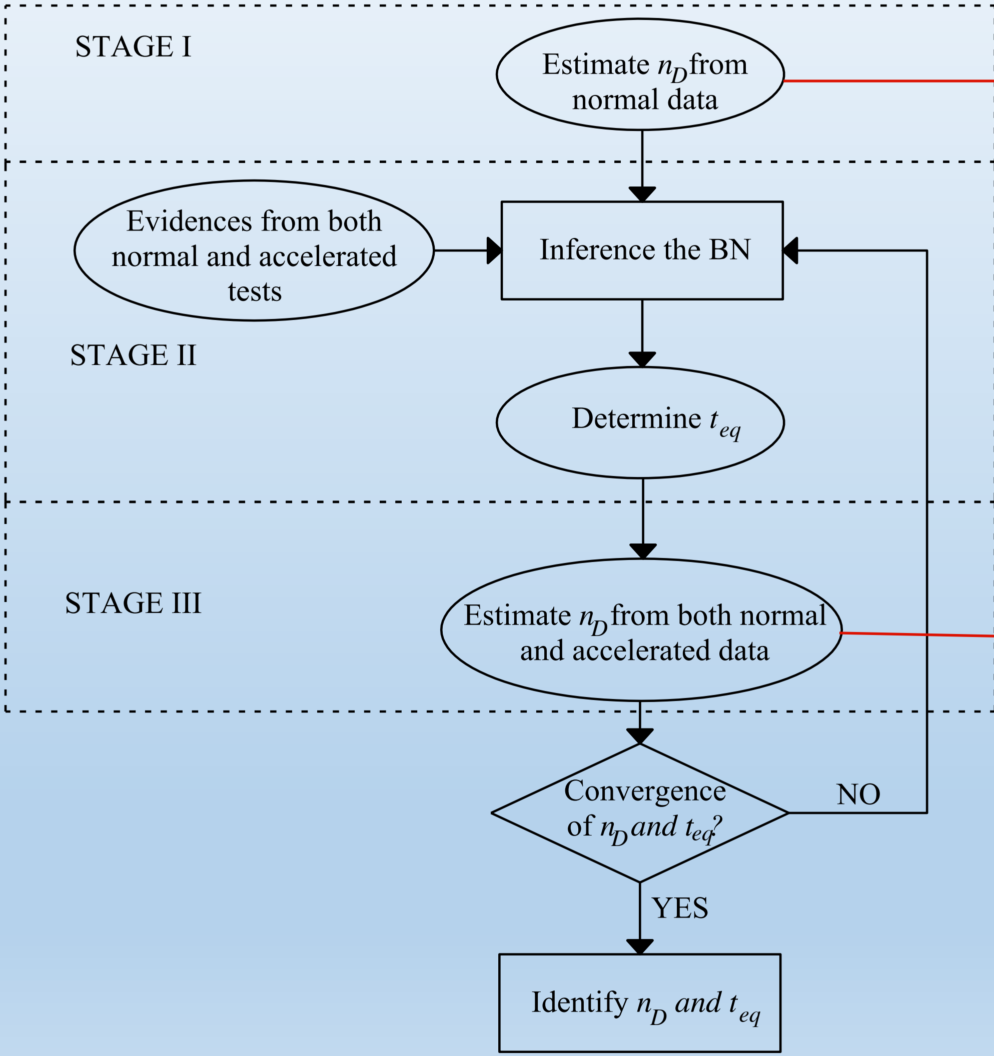
$$C(x,t) = C_s\left[1-erf\left(\frac{x}{2\sqrt{\frac{D_0}{1-n_D}\left[\left(1+\frac{t'_{ex}}{t}\right)^{1-n_D}-\left(\frac{t'_{ex}}{t}\right)^{1-n_D}\right]\left(\frac{t'_0}{t}\right)^{n_D} t}}\right)\right]$$

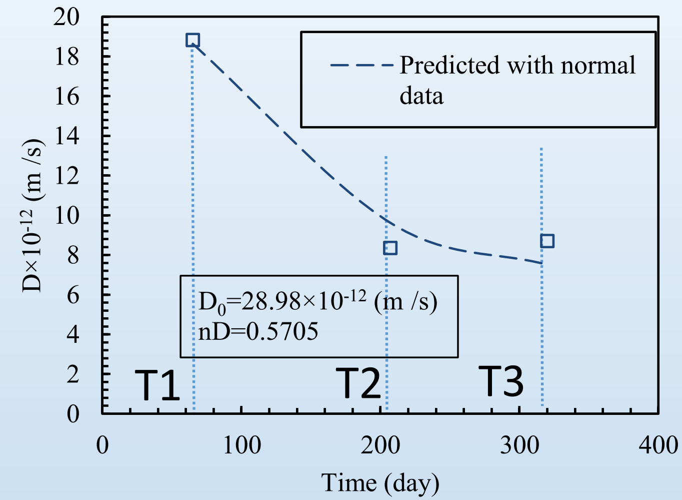
$$C(x_i,t_j) = f(x,t,C_s,D,n_D,...)$$

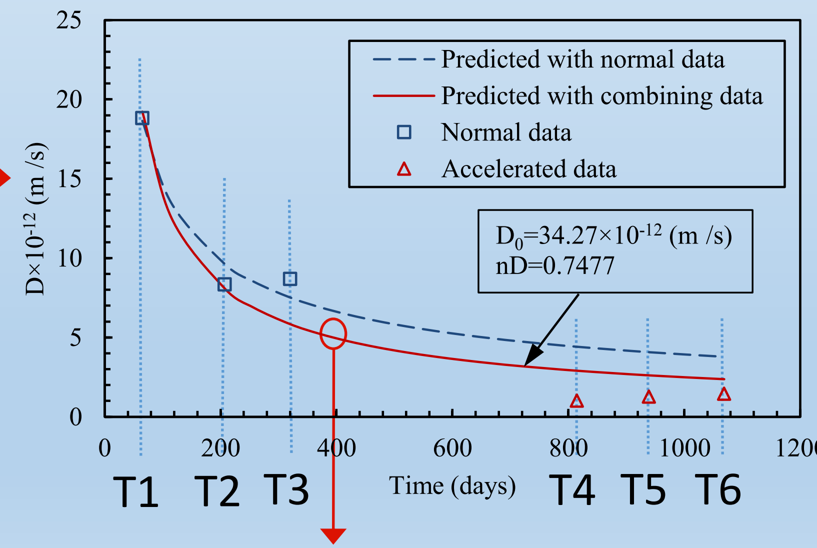


- The age factor n_D is **constant** → reduce uncertainties
- **Combining** information from **normal and accelerated tests** → addition parent node t_{eq}

❑ Proposed approach for estimating n_D and t_{eq}







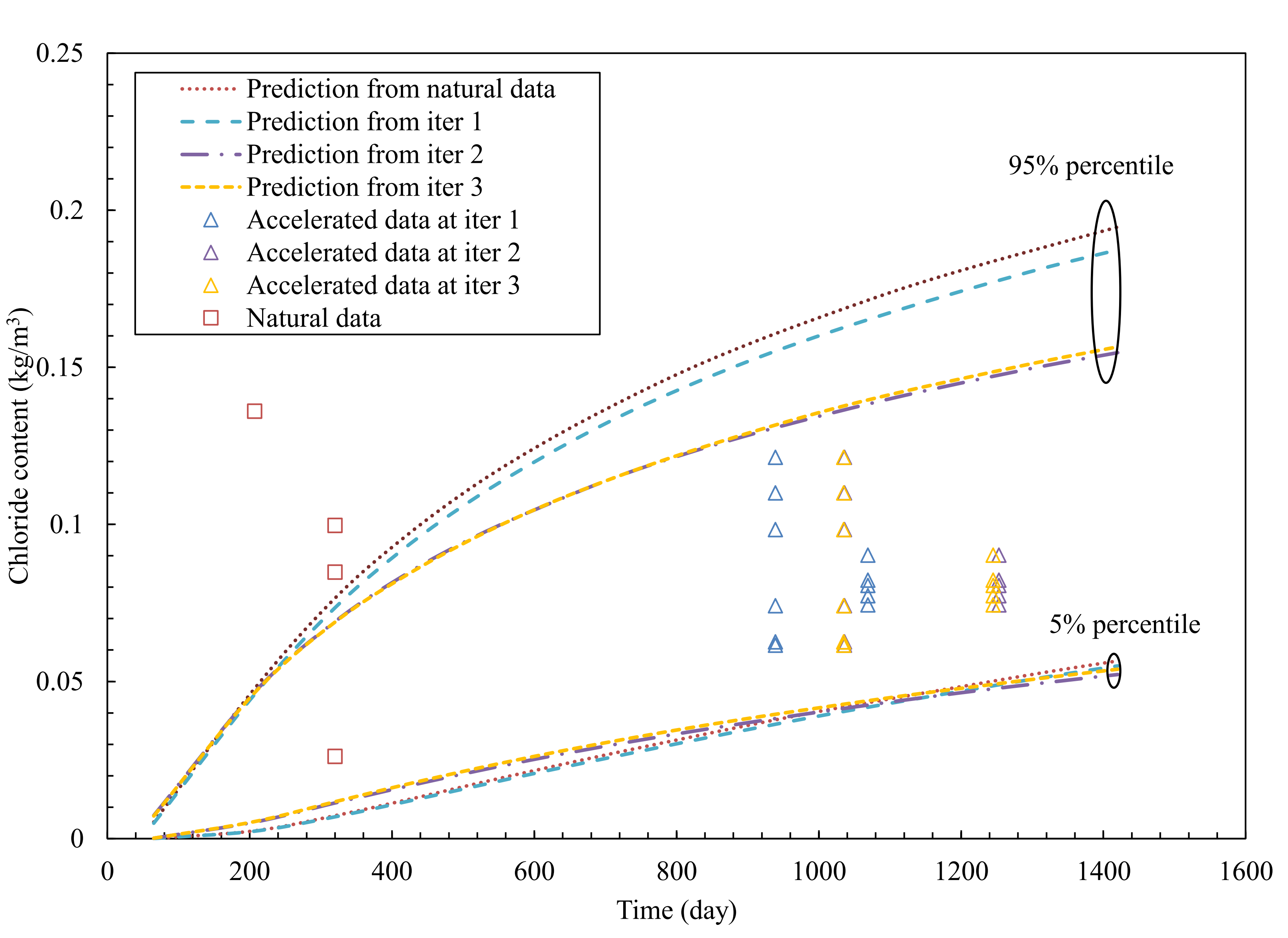
Better assessment of D(t)

Proposed approach for estimating n_D and t_{eq}

Evolution of model parameters after the iterative procedure

convergence

Parameter	Update with normal data	Iteration 1	Iteration 2	Iteration 3	Iteration 4
Mean C_s [kg/m³]	0.6048	0.5912	0.5712	0.5719	0.5719
Std C_s [kg/m³]	0.0999	0.0975	0.0920	0.0923	0.0923
Mean D_0 $\times 10^{-12}$[m²/s]	12.4	12.1	16.3	16.2	16.2
Std D_0 $\times 10^{-12}$ [m²/s]	4.91	4.80	5.09	5.09	5.10
n_D	0.5700	0.5700	0.7477	0.7398	0.7401
T4 [days]	–	815	899	893	893
T5 [days]	–	939	1036	1035	1035
T6 [days]	–	1069	1253	1245	1245



5% and 95 percentiles of chloride content at depth x=31.5mm.

Project ANR CLIMBOIS

Dynamic Bayesian Network for reliability assessment of degradation structures with consideration of spatial variability

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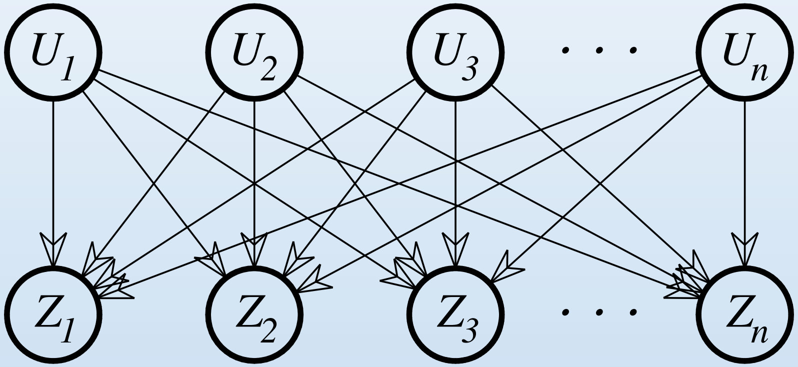
Modelling random field using BN

- Decomposes vector **Z** of correlated random variables

$$\mathbf{Z} = \mathbf{TU} = \begin{bmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{n1} & \cdots & t_{nn} \end{bmatrix} \begin{Bmatrix} U_1 \\ \vdots \\ U_n \end{Bmatrix}$$



Z: correlated standard normal random variables
T : transformation matrix
U: independent standard normal random variables

- Problems:

- BN with densely connected nodes
- Size of CPTs ↑
- Computational intractable

- Nodes and links elimination (Bensi 2011)
- Common Source Random Variables (Song and Kang 2009)

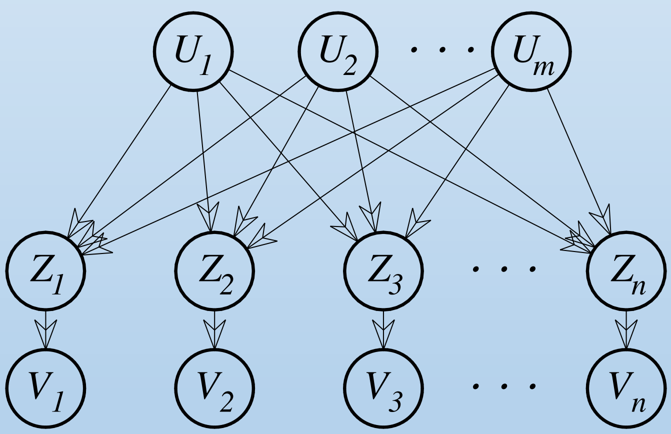
Modelling random field using BN

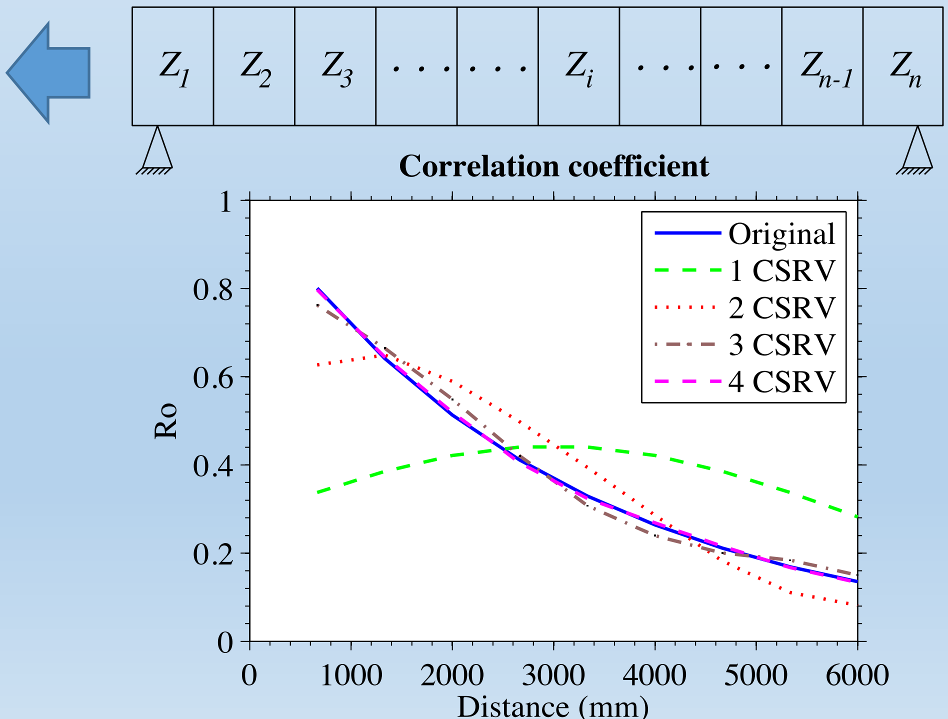
- Common Source Random Variables (CSRVs):

$$Z_i = \sqrt{1-\sum_{k=1}^{m} r_{ik}^2}.V_i + \sum_{k=1}^{m} r_{ik} U_k$$ with , U_k, k=1,…,m: **CSRVs**
i=1,…,n

r_{ik}: determine by solving optimizing problem:

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\rho_{ij} - \sum_{k=1}^{m} r_{ik} r_{jk} \right]^2$$ Subject to: $$\sum_{k=1}^{m} r_{ik}^2 \leq 1, i = 1,\ldots,n$$





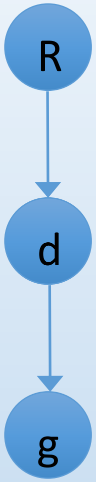
U,V: independent standard normal random variables

❑ DBN for modelling structural reliability of timber structures subjected to decay

- Decay deterioration

Decay model:
$$\begin{cases} r = k_{wood} . k_{climate} \\ t_{lag} = 8.5 r^{-0.85} \end{cases}$$

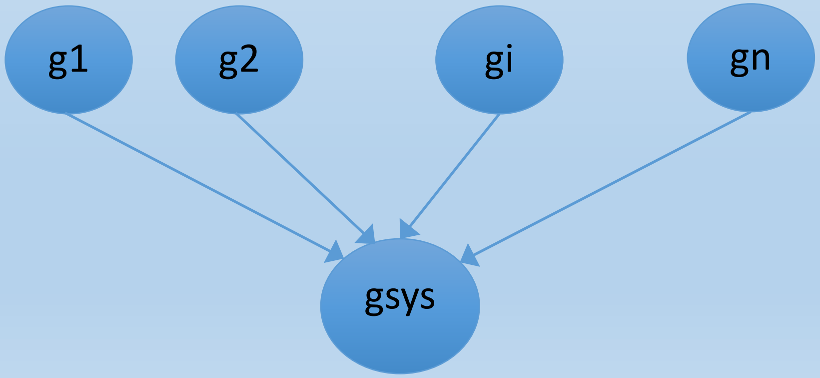
Decay depth:
$$d(t) = \begin{cases} 0 & t \le t_{lag} \\ r(t - t_{lag}) & t > t_{lag} \end{cases}$$



- Limit state function (replacement event)

Element: $g_i(t) = d_i(t) - 10mm$

System (series): $g_{sys}(t) = g_1(t) \cup g_2(t) \cup ... g_i(t) \cup ... g_n(t)$



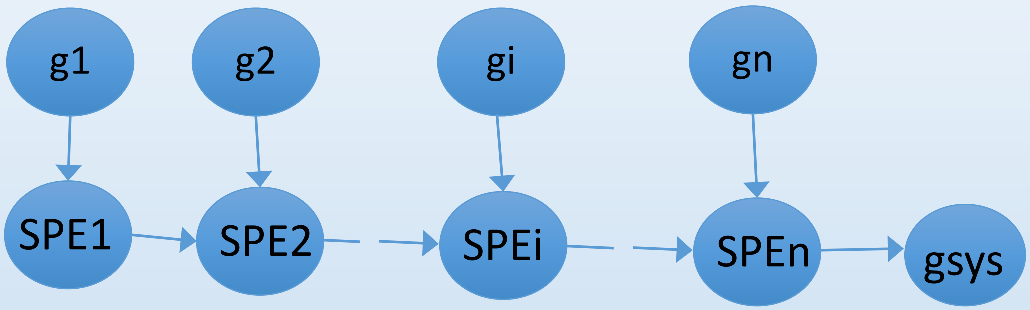
The CPT of g_{sys} has $\mathbf{2^{n+1}}$ entries

→ The CPT's size very large when n increase

→ Using Survival Path Event (SPE)

❑ Dynamic Bayesian Network for modelling structural reliability

- Modelling system performance with Survival Path Event (SPE)

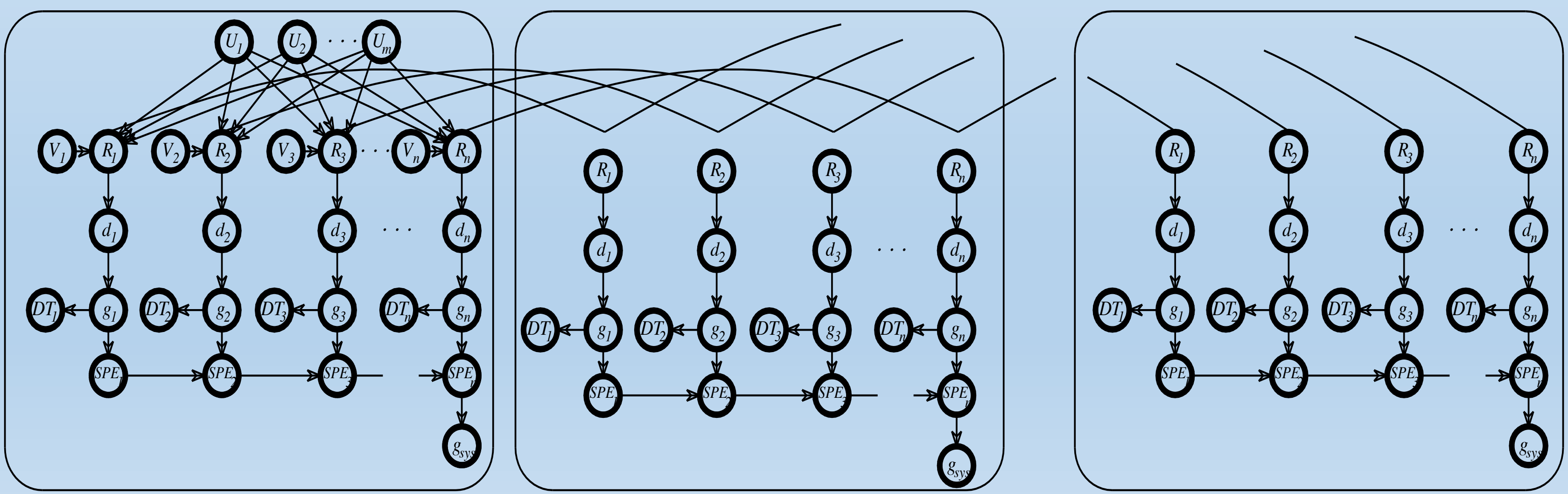


→ CPT of gsys has 2^2 entries

Problems and definition of SPE:

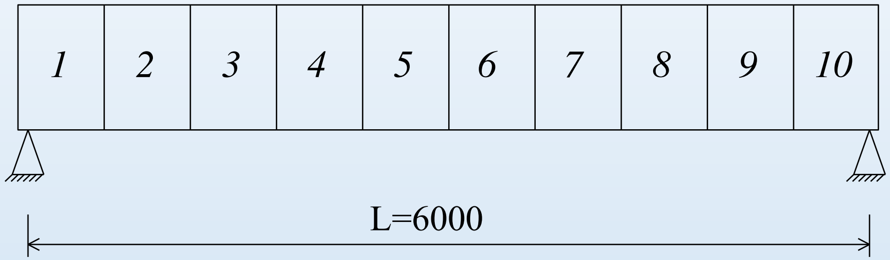
$$\begin{cases} \text{SPE}_i = \text{survival if } \{\text{SPE}_{i-1} = \text{survival}\} \cup \{g_i = \text{survival}\} \\ \text{SPE}_i = \text{failure otherwise} \end{cases}$$

- DBN configuration for modelling system performance

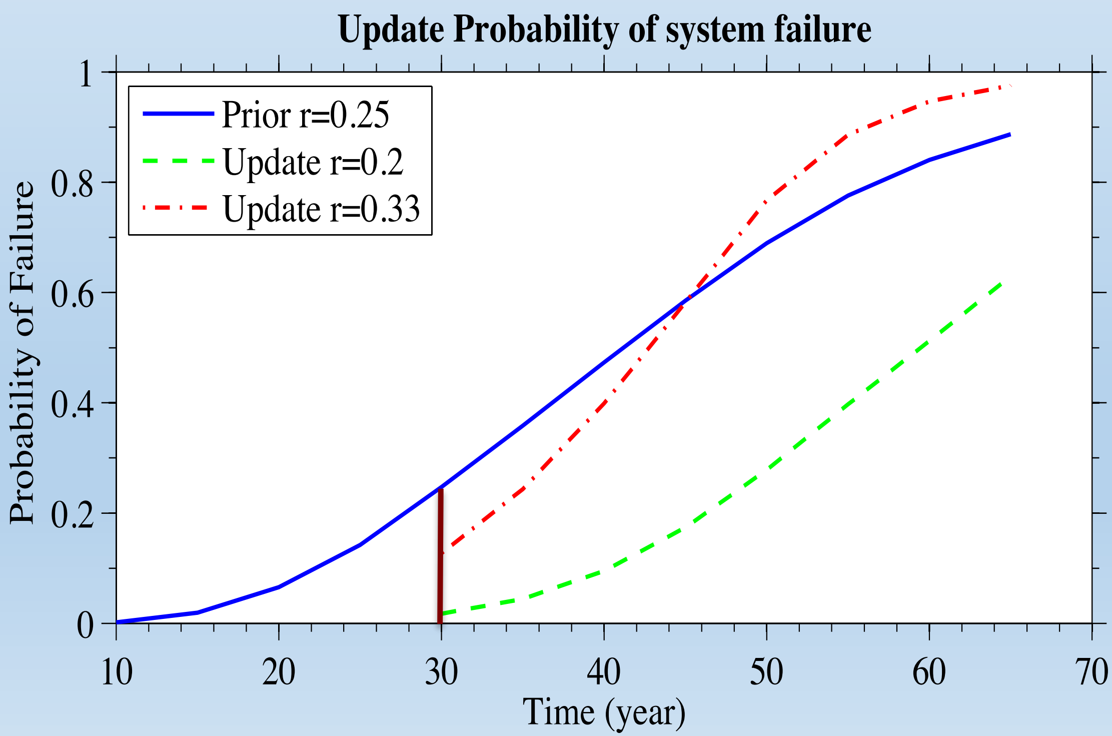


Slice $t = 0$ Slice $t = 1$ Slice $t = T$

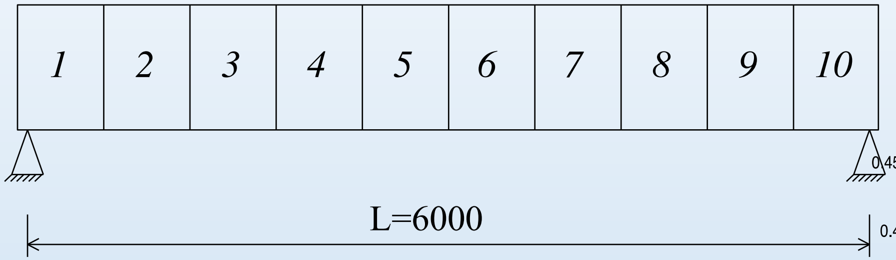
Updating structural reliability with inspection data



- Inspection at 30 years
- **Prior** : r = LN(0.25; COV=0.7)
- **Observation:**
  Case 1: r = LN(0.2; COV=0.3)
  Case 2: r = LN(0.33; COV=0.3)



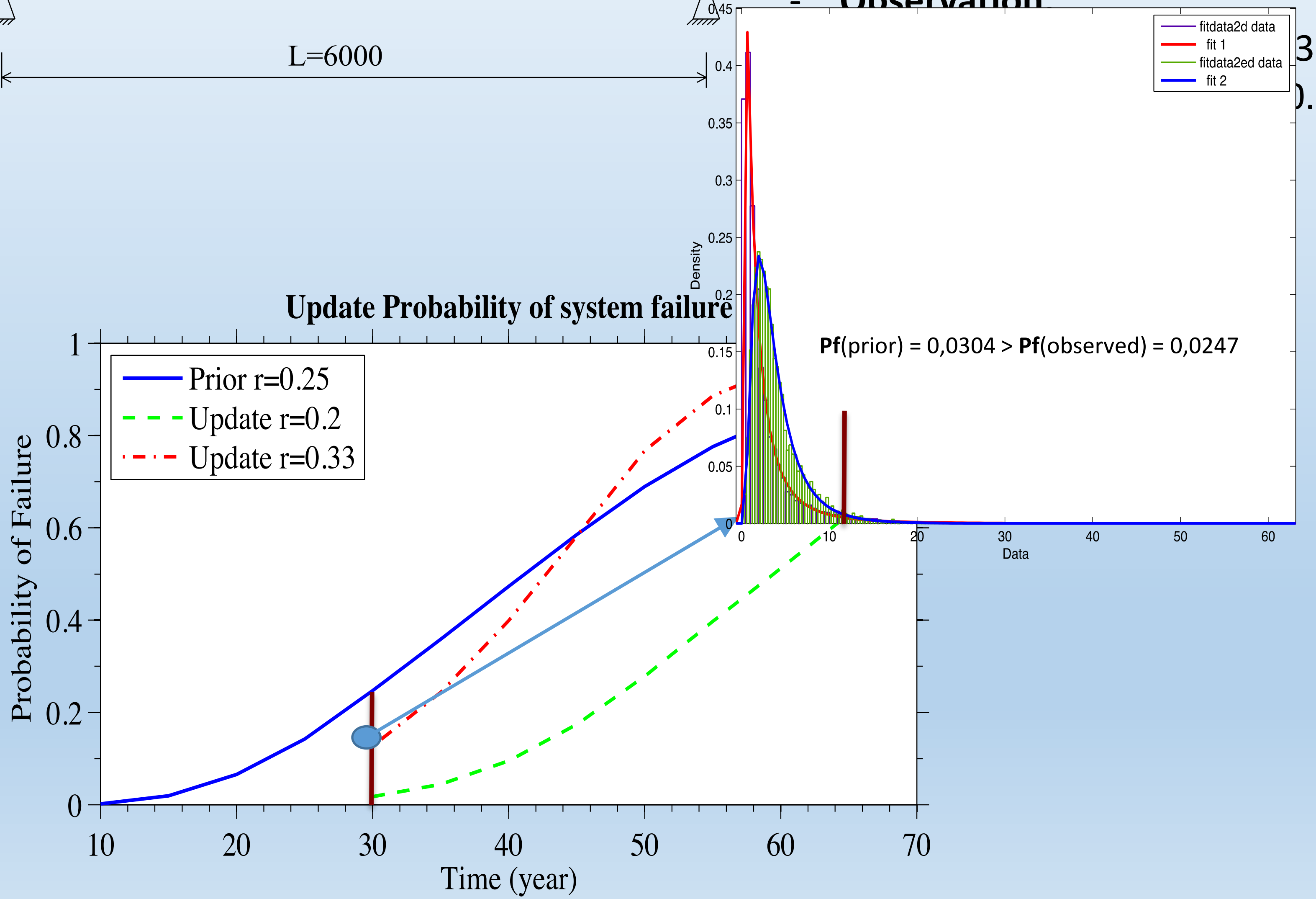
Updating structural reliability with inspection data



- Inspection at 30 years
- **Prior** : r = LN(0.25; COV=0.7)
- **Observation:**

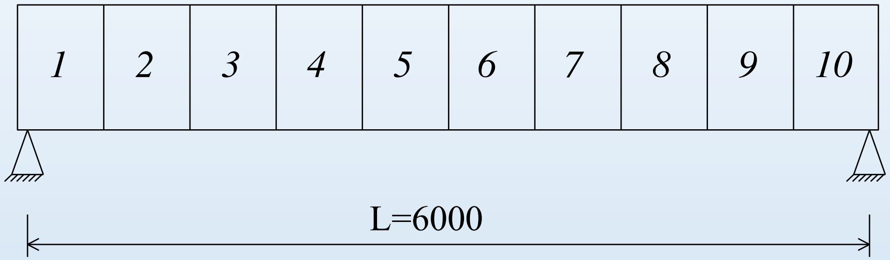
3)

).3)



Pf(prior) = 0,0304 > **Pf**(observed) = 0,0247

❑ Updating structural reliability with inspection data



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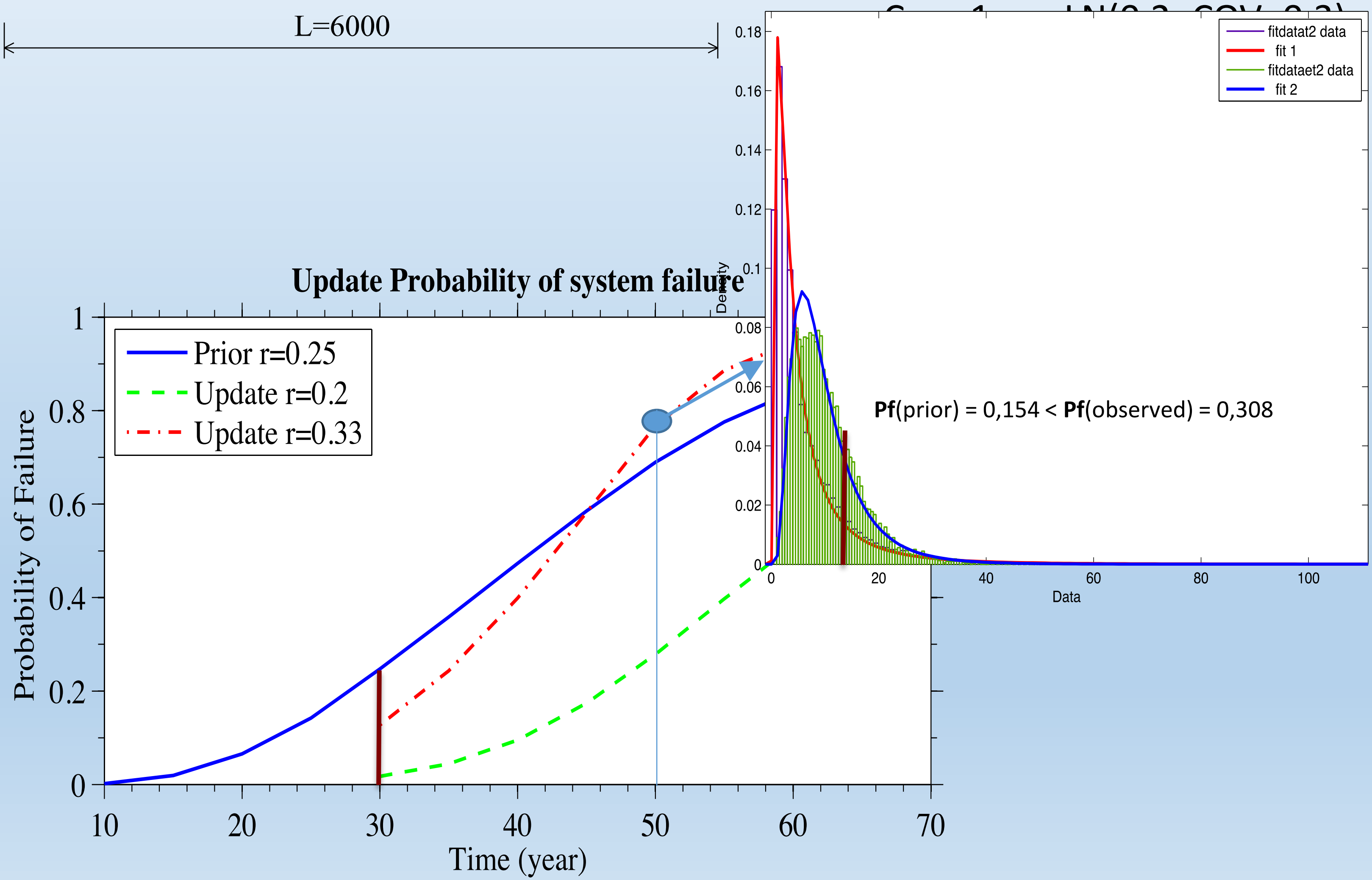

THANK YOU FOR YOUR ATTENTION